THE DESIGN OF LADDER NETWORKS CONTAINING LUMPED AND DISTRIBUTED ELEMENTS

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Abstract

A design procedure has been developed to design lumped/distributed ladder networks. The necessary and sufficient conditions on the coefficients of the characteristic polynomial have been obtained and a suitable low-pass to band-pass frequency transformation has been developed. The procedure was then applied to design a microwave filter and the results show good agreement with the theoretical predictions.

Introduction

General networks containing lumped and distributed elements have been dealt with by numerous authors. The work described in this paper deals with ladder networks and the main aim is to obtain a design procedure to realise practical circuits. The procedure has been applied to various examples with successful results.

The advantage of these circuits over circuits containing distributed elements only is that the response in the harmonic frequency bands can be greatly reduced and the advantage over circuits containing lumped elements only is that a greater rate of cut-off can be achieved for the same order because of the presence of finite transmission zeros.

Theoretical Analysis

A two variable prototype ladder network is shown in Figure 1. The impedances of all the series elements are proportional to the frequency variable $s = \sigma + j\omega$, and the admittances of all the shunt elements are proportional to another frequency variable $\lambda = \Sigma + j\Omega$. The variables s and λ are related by $\lambda = f(s)$ and in general f(s) can take various forms. In a prototype lumped/distributed network all the series elements are inductors and all the shunt elements are open-circuited lengths of commensurate transmission lines. In this case $f(s) = \tanh T_n s$ where T_n is the delay on each line.

The input impedance of the network in Figure 1a is given by

$$Z_{in}(s,\lambda) = \frac{c_o + c_1 s + c_2 s\lambda + c_3 s^2\lambda + \ldots c_n\, s^{\frac{n+1}{2}} \lambda^{\frac{n-1}{2}}}{d_o + d_1\lambda + d_2\lambda s + d_3\lambda^2 s + \ldots d_{n-1}\lambda^{\frac{n-1}{2}} s^{\frac{n-1}{2}}} \qquad (1)$$

for n odd, and

$$Z_{in}(s,\lambda) = \frac{c_o + c_1 s + c_2 s\lambda + c_3 s^2\lambda \ \ldots c_n\, s^{\frac{n}{2}} \lambda^{\frac{n}{2}}}{d_o + d_1\lambda + d_2\lambda s + d_3\lambda^2 s + \ldots d_{n-1}\lambda^{\frac{n}{2}} s^{\frac{n}{2}-1}} \qquad (2)$$

for n even.

Similar forms can be obtained for the input impedance of the network in Figure 1b.

Without loss of generality the constants c_o and d_o can be normalised to

$$c_o = R_2 \text{ and } d_o = 1 \qquad (3)$$

Conditions must exist on the constants c and d in order that the impedance function can be expanded in the continued fraction form with positive coefficients

$$Z_{in}(s,\lambda) = \alpha_1 s + \frac{1}{\beta_2\lambda} + \frac{1}{\alpha_3 s} + \ldots \frac{1}{\alpha_n s} + \frac{1}{R_2} \qquad (4)$$

The necessary and sufficient conditions that the required continued fraction form exists are given by

$$\left.\begin{aligned} c_1 d_1 &= c_o d_2 + c_2 d_o \\ c_1 d_3 + c_3 d_1 &= c_o d_4 + c_2 d_2 + c_4 d_o \\ &\ddots \\ c_n d_{n-2} &= c_{n-1} d_{n-1} \end{aligned}\right\} \qquad (5)$$

Conditions (5) together with (3) form n+1 conditions on the 2n+1 coefficients c and d. Thus n coefficients could be chosen and the remaining n+1 coefficients are then determined. In other words there are n conditions that can be imposed on the response of the network as in the case of a lumped ladder network.

The input scattering parameter $S_{11}(s,\lambda)$ can be obtained

$$S_{11}(s,\lambda) = \frac{a_o + a_1' s + a_1\lambda + a_2 s\lambda + a_3' s\lambda^2 + a_3 s^2\lambda + \ldots a_n s^{\frac{n+1}{2}} \lambda^{\frac{n-1}{2}}}{b_o + b_1' s + b_1\lambda + b_2 s\lambda + b_3' s\lambda^2 + b_3 s^2\lambda + \ldots b_n s^{\frac{n+1}{2}} \lambda^{\frac{n-1}{2}}} \qquad (6)$$

for n odd, and a similar expression can be obtained for n even.

All the odd terms except the last are split into two terms and the total number of either the a or the b constants is $\frac{3n+1}{2}$ for n odd and $\frac{3n+2}{2}$ for n even.

The transfer scattering parameter $S_{21}(s,\lambda)$ is given by

$$S_{21}(s,\lambda) = \frac{2\sqrt{R_1 R_2}}{b_o + b_1' s + b_1\lambda + b_2 s\lambda + b_3' s\lambda^2 + b_3 s^2\lambda + \ldots b_n s^{\frac{n+1}{2}} \lambda^{\frac{n-1}{2}}} \qquad (7)$$

The relations between the b constants in (7) and the c and d constants in (1) are given by

$$\begin{aligned} b_i &= c_i + d_i && \text{for i even} \\ b_i' &= d_i && \multirow{2}{*}{} \text{for i odd} \\ b_i &= c_i && \end{aligned} \qquad (8)$$

The b constants in the characteristic polynomial must satisfy the conditions in (8) and the c and d constants in turn must satisfy the conditions in (5).

Frequency Transformation

After the low-pass prototype network is obtained a frequency transformation step is required such that the relation

$$H_n(\omega_n, \Omega_n) = H(\omega, \Omega) \tag{9}$$

is satisfied, where the subscript n refers to the normalised quantities.

For the lumped/distributed case we also have

$$\Omega_n = \tan T_n \omega_n \text{ and } \Omega = \tan T\omega \tag{10}$$

To obtain the required transformation the relations between the normalised and denormalised frequencies are

$$\omega_n = f_1(\omega) \text{ and } \Omega_n = f_2(\Omega) \tag{11}$$

Furthermore, if the resulting network is to be realisable both $f_1(\omega)$ and $f_2(\Omega)$ must be positive-real functions.

From (10) and (11) we have

$$f_2(\Omega) = \tan\left[T_n\ (f_1(\omega))\right] = \tan\left[T_n f_1\ (\tfrac{1}{T}\tan^{-1}\Omega)\right] \tag{12}$$

Thus in order that (9) is satisfied the functions f_1 and f_2 are related by (12). It is not always possible to obtain positive-real functions f_1 and f_2 such that (12) is satisfied and (9) is satisfied for all values of ω.

Frequency scaling. Scaling the frequency by a factor n_f can be easily achieved by the relations

$$\begin{aligned} \omega &= n_f\omega_n \\ \lambda &= \tan(n_f T_n \omega) = \tan T\omega \end{aligned} \tag{13}$$

All the values of the series inductors are divided by n_f and the commensurate delay T of the scaled network is $n_f T_n$.

In this case both (9) and (12) are satisfied and the transformation is valid for all values of ω.

Low-pass to band-pass transformation. In this case it is not possible to find two positive-real functions f_1 and f_2 to satisfy the above conditions. The alternative is to satisfy (9) at a number of discrete frequencies. This could be achieved by choosing

$$f_1(\omega) = \frac{\omega_{nc}\omega_o}{\omega_2-\omega_1}\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)$$

$$\text{and}\quad f_2(\Omega) = \frac{\Omega_{nc}\Omega_o}{\Omega_2-\Omega_1}\left(\frac{\Omega}{\Omega_o} - \frac{\Omega_o}{\Omega}\right) \tag{14}$$

where ω_{nc} is the normalised cut-off frequency, $\omega_o^2 = \omega_1\omega_2$, ω_1 and ω_2 are the cut-off frequencies of the band-pass response, $\Omega_{nc} = \tan T_n\omega_n$, $\Omega_1 = \tan T\omega_1$, $\Omega_2 = \tan T\omega_2$ and $\Omega_o^2 = \Omega_1\Omega_2 = (\tan T\omega_o)^2$.

The condition $\Omega_o^2 = \Omega_1\Omega_2$ will determine the required value of T, the commensurate delay of the band-pass network. This is obtained by solving the equation

$$\tan T\omega_1\ \tan T\omega_2 = \tan^2 T\omega_o \tag{15}$$

There are several solutions to (15). Each solution will give a set of transformed impedance values and the final choice of T will be such that the most practical impedance values are obtained.

The resulting impedance transformations of the circuit elements are shown in Figure 2.

The above transformation ensures that (9) is satisfied only at (ω_n=o, $\omega=\omega_o$), ($\omega_n=\omega_{nc}$, $\omega=\omega_1$) and ($\omega_n=\omega_{nc}$, $\omega=\omega_2$).

The normalised cut-off frequency ω_{nc} can be chosen as either the 3dB cut-off frequency or the transmission zero frequency or any other convenient value.

Practical Examples

Example I: A fifth order low-pass equiripple filter was designed with 0.5dB pass-band ripple and a cut-off frequency of 3GHz.

First the b constants were determined for the normalised prototype with T_n=1.2 to satisfy the equiripple criteria. The circuit elements were then calculated for the required filter.

Figure 3 shows the final circuit. The theoretical response and measured points are shown in Figure 4.

Example II: A lumped/distributed third order, maximally flat, ladder filter is designed to meet the following specifications.

$R_1 = R_2 = 50\Omega$ and the 3dB cut-off frequencies are at 2.823GHz and 3.183GHz (f_o = 3GHz).

The b constants were determined for T_n=1 and the elements of the low-pass prototype were then calculated.

Equation (15) was then solved for T and the value of T for the most practical values of circuit elements was found to be T=0.20485 ns.

Figure 5 shows the final band-pass circuit and a photograph of the actual filter constructed in microstrip form. The lumped inductance was wire wound and the lumped capacitor had an interdigital form.

Figure 6 shows the theoretical response of the filter and the measured points.

The main disadvantage of the lumped/distributed circuit is the practical difficulties encountered in designing and producing the lumped elements at microwave frequencies. Various empirical formulae exist[1,2] for the design of these elements but none are accurate enough for the accurate prediction of their performance. Furthermore, lumped elements produce higher losses than distributed ones. The lumped elements are believed to be responsible for the insertion loss at the center frequency of the filter and for the slight shift in the cut-off frequencies. The measured attenuations in the harmonic bands are generally lower than the theoretical values; this is thought to be due to the coupling between the elements which exists in microstrip circuits.

Conclusion

A successful design procedure has been developed for the design of lumped/distributed ladder networks. These networks have useful applications in the lower microwave range where the lumped elements can have reasonable performance. More accurate design procedure for lumped elements will be required if high order filters of this type are to be designed.

References

1. C.S.R. Aitchinson, et al, "Lumped-circuit Elements at Microwave Frequencies," IEEE Trans., MTT, pp. 928-937, 1971.

2. M. Caulton, "The Lumped Element Approach to Microwave Integrated Circuits," Microwave Journal, pp. 51-58, 1970.

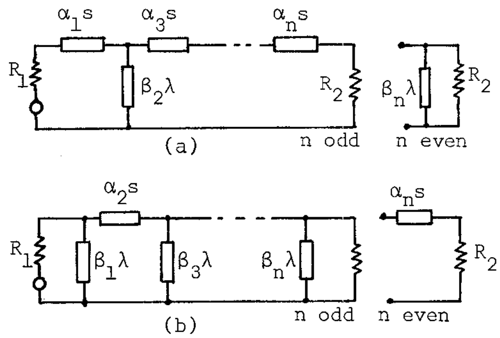


Fig. 1-Lumped/distributed ladder network.

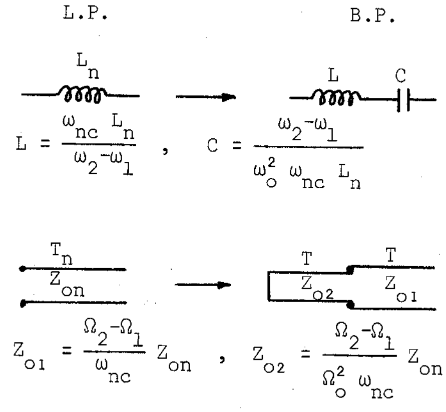
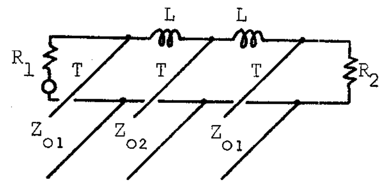


Fig. 2-Low-pass to band-pass transformations.



$R_1 = R_2 = 50\Omega$

$T = 0.06366$ ns

$Z_{o1} = 56.37\Omega$ $Z_{o2} = 34.44\Omega$

$L = 2.433$ nH

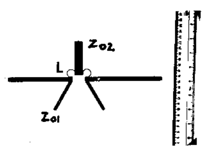


Fig. 3-Final circuit of fifth order filter (Example I).

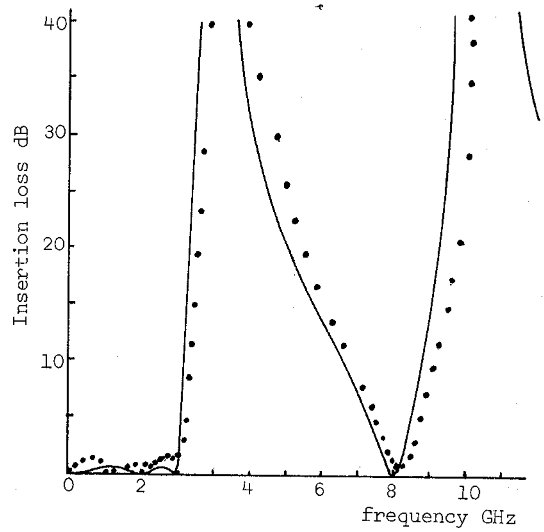
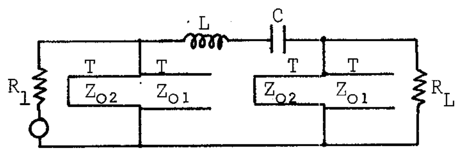


Fig. 4-Theoretical response and measured points of fifth order Chebyshev filter (Example I).



$R_1 = R_2 = 50\Omega$

$T = 0.20485$ ns

$Z_{o1} = 44.2\Omega$ $Z_{o2} = 57.5\Omega$

$L = 44.2$ nH $C = 0.06$ pF

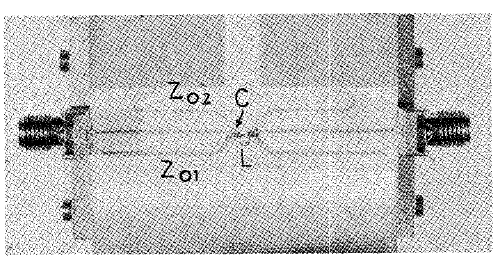


Fig. 5-Final band-pass circuit (Example II).

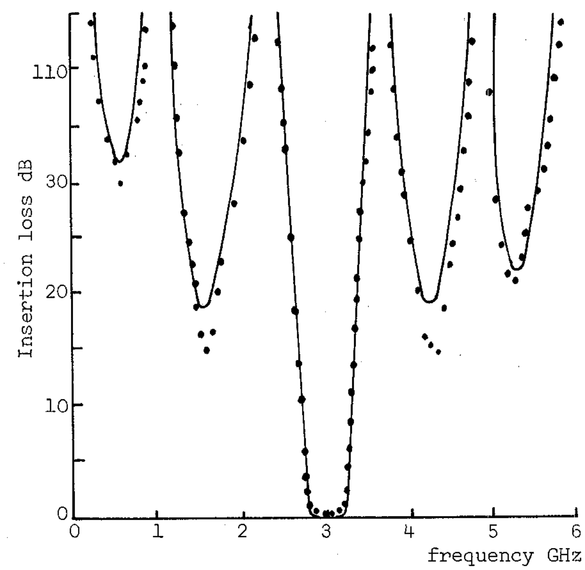


Fig. 6-Theoretical response and measured points of third order maximally flat band-pass filter (Example II).